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Finding the inversion temperature for water evaporation into an air–steam mixture

E.P. Volchkov [a], A.I. Leontiev [b], S.N. Makarova [a,*]

[a] *Kutateladze Institute of Thermophysics, SB RAS, 1 Lavrentyev Avenue, Novosibirsk 630090, Russia*
[b] *Moscow State Technical University of the Bauman, 5 Second Bauman avenue, Moscow 107005, Russia*

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Abstract

This paper deals with adiabatic evaporation of water into an air–steam mixture and pure superheated steam. The focus is made on the inversion temperature, which means that the rate of liquid evaporation into superheated steam becomes equal to the rate of evaporation into dry air. A simple analytical solution for the inversion temperature was derived. The analytical and numerical methods were applied for analysis of different factors (vapor quantity, flow rate, flow regime) on the value of inversion temperature.

Keywords: Inversion temperature; Evaporation; Superheated steam

1. Introduction

The advantage of drying with superheated steam instead of dry air was known for last 50 years [1,2]. For today, this method of material treatment is widely used in different industries (food processing, textile, etc.). The finding of optimal modes is an important engineering problem in developing of drying technologies for different materials. It was pointed out in one of pioneer papers of liquid conversion into superheated steam [1] that liquid evaporation into vapor can be more intensive than into dry air if the mainstream temperature exceeds the "critical" temperature. This temperature was named by Yoshida and Hyodo in 1970 [3] the inversion temperature (Fig. 1). The experimental value of inversion temperature was 170 °C for water evaporating from a wet vertical column into a turbulent flow. Several parameters have the influence on this temperature: gas flow rate (steady in volume or in mass), flow regime (laminar or turbulent) and vapor quantity in the flow.

Most of publications on this subject (experimental or numerical research) deal with water evaporation into the air–steam flow; this case is most close to applications. By the range of obtained inversion temperature is rather wide: from 140 °C at a constant mass flow rate [4] up to 390 °C at a constant volumetric flow rate of air–steam mix [5].

In experimental works, usually the mass flow rate of the drying gas was taken constant. It was shown [3,5,6] that the flow rate value is insignificant for the inversion temperature. Schwartze and Bröcker [5] made analytic calculations for water evaporating on a vertical wet column into a turbulent boundary layer (for the cases of gas flow rate constant in mass and volume). The important result was that the inversion temperature at a constant mass flow rate (198.6 °C) is very different from the value at a constant volumetric flow rate (390 °C). As for the influence of flow regime (laminar – $t_{\text{inv}} = 200, \ldots, 260$ °C, turbulent – $t_{\text{inv}} = 170, \ldots, 220$ °C) and geometry of evaporation surface on this temperature is not serious [5–9]. The influence of vapor quantity of the drying agent is a controversial issue – the late publications [3,6] demonstrates that there is no such effect, but the earlier publications [4,5,8–10] demonstrated an opposite result, i.e., the every level of humidity has its own level of inversion temperature.

The pioneer research by Costa and Neto da Silva [10] gave the formula for the inversion temperature for the case

* Corresponding author. Tel.: +7 383 3391041; fax: +7 383 3308480.
E-mail address: snmakarova@itp.nsc.ru (S.N. Makarova).

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Nomenclature

c_p	specific heat capacity at constant pressure
D	diffusion coefficient
h	enthalpy
j	transversal mass flux
K	mass concentration of the mixture component
Le	Lewis number
M	molar mass
P	pressure
Pr	Prandtl number
q	heat flux density
r	heat of vaporization
Re	Reynolds number
Sc	Schmidt number
St	Stanton number
t	temperature (°C)
T	temperature (K)
u, v	longitudinal and transversal velocity components
x, y	longitudinal and transversal coordinates relative the plate

Greek symbols

δ	boundary layer thickness
λ	thermal conductivity
μ	dynamic viscosity
ρ	density

Subscripts and superscripts

0	conditions on the outer side of boundary layer
a	dry air
mix	air–steam mixture
s	saturation conditions
t	turbulent
v	water vapor
w	wall conditions

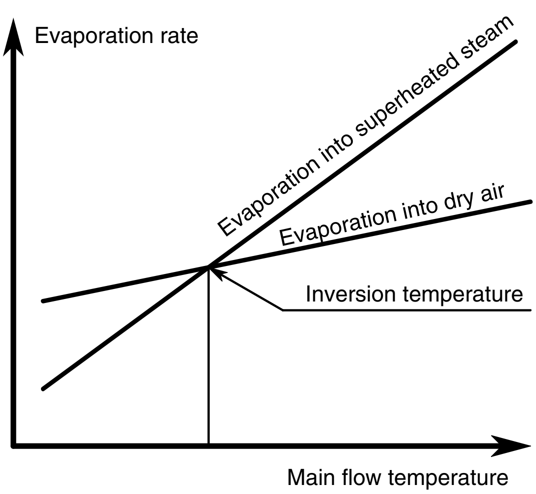

Fig. 1. About the inversion temperature.

of a liquid on a wet vertical column evaporating into a turbulent flow of air–steam mixture. But we are going to demonstrate that the function developed by Costa is not unique. Depending on the specific of the evaporation process, we obtained different relationships describing the inversion temperature.

The authors have used simple integral relations for finding the simple formulas for inversion temperature during water evaporation from a flat plate. We analyze the effect of gas flow rate of main stream (constant volumetric rate, constant mass rate), flow regime (laminar, turbulent), and composition of air–steam mixture on the inversion temperature. The evaluation of accuracy by the formula can be made via comparison with data of the numerical calculations.

2. Analytic study

The inversion temperature t_{inv} corresponds to condition $j_w^v = j_w^a$, so we considered in parallel two problems for adiabatic evaporation of liquid into a boundary layer of superheated steam (of same liquid) and into boundary layer of dry air. The flow diagram passing a flat plate is shown in Fig. 2.

For adiabatic evaporation, all heat flux from gas to liquid is consumed for evaporation. We can write down the heat flux towards the wall during liquid evaporation into superheated vapor:

$$q_w^v = j_w^v r^v; \tag{1}$$

and into dry air:

$$q_w^a = j_w^a r^a, \tag{2}$$

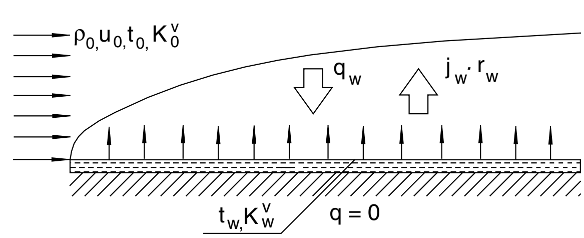

Fig. 2. The schematic of the mathematical model.

where j_w^v, j_w^a is the mass rate of evaporation into superheated vapor and dry air, correspondingly, r^v, r^a is the heat of vaporization at t_w^v, t_w^a , correspondingly.

We obtain from (1) and (2) the ratio of liquid mass flow on the wall:

$$\frac{j_w^v}{j_w^a} = \frac{q_w^v}{q_w^a}\frac{r^a}{r^v}. \tag{3}$$

On another hand, the heat flux on the wall in nonuniform boundary layer can be written as

$$q_w = -\frac{\lambda_w}{c_{pw}}\left(\frac{\partial h}{\partial y}\right)_w + \left(1 - \frac{1}{Le_w}\right)\rho_w D_w\left(\frac{\partial K}{\partial y}\right)_w (h_1 - h_2)_w.$$

Obviously, at $Le_w \to 1$ the heat flux $q_w \to -\frac{\lambda_w}{c_{pw}}\left(\frac{\partial h}{\partial y}\right)_w$. Since the Lewis number for water ranges from 1.2 to 1.8, this is a good approximation. Moreover, it was demonstrated in [11] that at $Le_w \to 1$

$$St_\lambda = \frac{-\lambda_w\left(\frac{\partial T}{\partial y}\right)_w}{\rho_0 u_0 c_{p0}(t_w - t_0)} = St_h = \frac{-\frac{\lambda_w}{c_{pw}}\left(\frac{\partial h}{\partial y}\right)_w}{\rho_0 u_0 (h_w - h_0)}.$$

Therefore, with the definition for Stanton thermal number we obtain the formula for heat flux

at evaporation into vapor:

$$q_w^v = St_\lambda^v \rho_0^v u_0^v c_{p0}^v (t_0^v - t_w^v) \tag{4}$$

and at evaporation into air:

$$q_w^a = St_\lambda^a \rho_0^a u_0^a c_{p0}^a (t_0^a - t_w^a), \tag{5}$$

where t_w^v, t_w^a is the wall temperature while evaporation into vapor and air, correspondingly.

Taking (3) and expressions for heat flux (4) and (5), we obtain:

$$\frac{j_w^v}{j_w^a} = \frac{St_\lambda^v \rho_0^v u_0^v c_{p0}^v (t_0^v - t_w^v) r^a}{St_\lambda^a \rho_0^a u_0^a c_{p0}^a (t_0^a - t_w^a) r^v}, \tag{6}$$

where the Stanton thermal number is found through the heat transfer law in the form:

$$St_\lambda = ARe_x^{-m} Pr_0^{-n} \Psi.$$

The relative function of heat transfer that accounts for the transversal mass flux is as follows for the given conditions:

$$\Psi = (St_\lambda / St_0)_{Re_x = \text{const}},$$

where $St_0 = ARe_x^{-m} Pr_0^{-n}$ is the Stanton number under "standard conditions". The laminar boundary layer has $n = 2/3$, $m = 0.5$; and the turbulent layer $n = 0.6$, $m = 0.2$.

Assuming the temperature of main stream to be the same for both variants of evaporation and equal $t_0 = t_0^v = t_0^a = \text{const}$, we obtain from (6) the relation of evaporation mass rates at a constant mass flow rate of gas ($\rho_0 u_0 = \rho_0^v u_0^v = \rho_0^a u_0^a = \text{const}$):

$$\frac{j_w^v}{j_w^a} = \frac{c_{p0}^v}{c_{p0}^a}\frac{(t_0 - t_w^v)}{(t_0 - t_w^a)}\frac{r^a}{r^v}\left(\frac{Pr_0^a}{Pr_0^v}\right)^n \frac{\Psi^v}{\Psi^a}. \tag{7}$$

The formula (7) is valid at the same Reynolds numbers $Re_x = \text{const}$, but many researchers compared the rate of evaporation into superheated vapor and air at the same distance from the front edge of plate x. Then the formula (6), taking the definition of the Reynolds number, gives us the proportion of evaporation mass rates at a constant mass flow rate ($\rho_0 u_0 = \rho_0^v u_0^v = \rho_0^a u_0^a = \text{const}$) and at $x = \text{const}$:

$$\frac{j_w^v}{j_w^a} = \frac{c_{p0}^v}{c_{p0}^a}\frac{(t_0 - t_w^v)}{(t_0 - t_w^a)}\frac{r^a}{r^v}\left(\frac{Pr_0^a}{Pr_0^v}\right)^n \left(\frac{\mu_0^v}{\mu_0^a}\right)^m \frac{\Psi^v}{\Psi^a}, \tag{8}$$

where μ_0^v, μ_0^a is the coefficient of dynamic viscosity of the main stream (two variants of evaporation mode).

In similar way, we can obtain the formula for inversion temperature at equal velocities of the main stream ($u_0 = u_0^v = u_0^a = \text{const}$) through the ideal gas law:

$$\frac{j_w^v}{j_w^a} = \frac{c_{p0}^v}{c_{p0}^a}\frac{(t_0 - t_w^v)}{(t_0 - t_w^a)}\frac{r^a}{r^v}\left(\frac{Pr_0^a}{Pr_0^v}\right)^n \left(\frac{\mu_0^v}{\mu_0^a}\right)^m \left(\frac{M^v}{M^a}\right)^{1-m} \frac{\Psi^v}{\Psi^a}, \tag{9}$$

where M^v, M^a are the molar mass of vapor and air, correspondingly.

If we consider instead of superheated vapor ($K_0^v = 1$) an air–steam mixture ($0 < K_0^v < 1$), the inversion temperature t_{inv} is found from the condition $j_w^{mix}/j_w^a = 1$.

The wall temperature during evaporation into an air–steam mixture or dry air can be calculated using the similarity of processes of heat and mass transfer jointly with the saturation curve. The heat- and mass-transfer similarity is usually conveyed through a function of Lewis number [11] $St_\lambda/St_D = Le^n$. The saturation curve for water vapor is described by well-known Antuan's equation [12]:

$$P_S = 133.322\ \exp\left(18.3036 - \frac{3816.44}{T - 46.13}\right).$$

It was demonstrated [13] that for analysis of heat and mass transfer in a boundary layer with varying composition the Lewis number should be calculated from the wall parameters (or on the liquid film for the case of evaporation).

Taking the definitions of the thermal and diffusive Stanton number, the similarity of laws of heat and mass transfer can be reduced to the form:

$$\frac{r^a}{c_{p0}^a (t_0 - t_w^a)} = Le_w^n \frac{1 - K_w^a}{K_w^a - K_0^a}. \tag{10}$$

The wall temperature calculations for water evaporation into dry air by formula (10) in compare with numerical calculations and experimental results [13–16] are plotted in Fig. 3. Obviously, calculations by formula (10) are close to numerical results and (within the error interval) to the experimental data.

The relative laws of heat transfer in expressions (7)–(9) Ψ^v/Ψ^a for laminar flow can be written down using the film flow theory [11]:

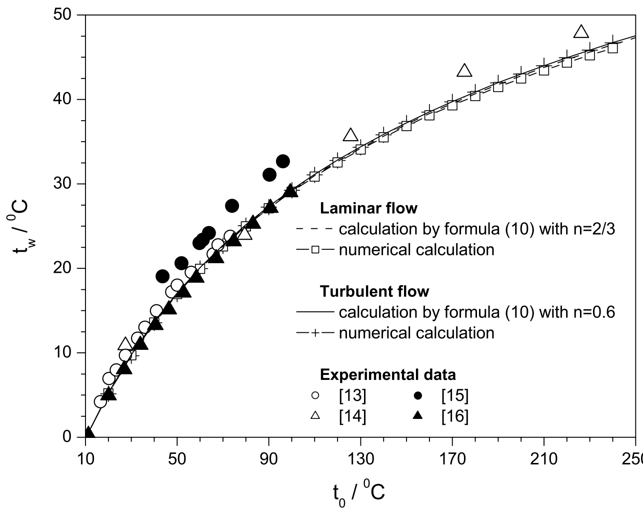

Fig. 3. Surface temperature for water evaporation into dry air.

$$\Psi = \sqrt{\frac{\Psi_{Re^{**}}}{1+b_{T1}}}, \quad \Psi_{Re^{**}} = \frac{\ln(1+b_{T1})}{b_{T1}},$$

where $b_{T1} = c_{p0}(t_0 - t_w)/r$ is the thermal parameter of permeability.

For a turbulent flow, the theory of limiting relative laws is valid [17]:

$$\Psi = \left(\frac{2}{\sqrt{1+b_{T1}}+1}\right)^{1.6}.$$

Thus, the obtained analytical relations (7)–(9) are suitable for calculation of inversion temperature in the case of a liquid evaporating from a flat plate at different parameters of the gas flow.

3. Numerical study

Problem statement. The flow can be described by equations for steady two-dimensional boundary layer of a binary gas mix (water vapor (v) and air (a)):

the continuity equation

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{11}$$

the motion equation

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left((\mu+\mu_t)\frac{\partial u}{\partial y}\right), \tag{12}$$

the energy equation

$$\rho u c_p \frac{\partial T}{\partial x} + \rho v c_p \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left((\lambda+\lambda_t)\frac{\partial T}{\partial y}\right) + \left(\frac{\mu}{Sc}+\frac{\mu_t}{Sc_t}\right)(c_p^v - c_p^a)\frac{\partial K^v}{\partial y}\frac{\partial T}{\partial y}, \tag{13}$$

the diffusion equation

$$\rho u \frac{\partial K^v}{\partial x} + \rho v \frac{\partial K^v}{\partial y} = \frac{\partial}{\partial y}\left((\rho D + \rho D_t)\frac{\partial K^v}{\partial y}\right), \tag{14}$$

$$K^a = 1 - K^v.$$

The boundary conditions for the system of differential equations (11)–(14) are written in the form:

– On the wall $y = 0$:

$$u(0) = 0, \left(\lambda\frac{\partial T}{\partial y}\right)_w = j_w \cdot r_w, \quad K^v(0) = K_w^v,$$
$$K^a(0) = 1 - K_w^v;$$

– On the outer side of the boundary layer $y = \delta$:

$$u(\delta) = u_0, \quad T(\delta) = T_0, \quad K^v(\delta) = K_0^v, \quad K^a(\delta) = 1 - K_0^v.$$

The turbulent flow characteristics were calculated through following models:

- Cebeci algebraic model modified by Landis and Mills [18];
- model by Lam–Bremhorst [19].

Method of solution. We used the method for solving of boundary layer equations with $X - \omega^2$ coordinate transformation developed by Patankar and Spalding [20] and modified by Denny and Mills [21]. The discretization was carried out through the method of indefinite coefficients by the Crank–Nicolson scheme. The resulting system of linear equations can be written in a form of three-diagonal matrix and solved by the Tomas method (sweep method) described in [20]. The nonlinearity of differential equations was removed by simple iteration method on every step of integration with the accuracy of 10^{-5}%. The grid compression was applied near the wall. The mesh point number varied from 50 to 100 over the boundary layer thickness. The step by x axis varied as a function of the current length of the dynamic boundary layer by the law $\Delta x = \delta/100$.

4. Results and discussion

Fig. 4 shows the comparison of calculations by formula (8) and results of the numerical calculations and data from other papers [5–9]. It is remarkable that for the laminar flow the calculations by our formula are closer to experiment and numerical data than previous calculations [6]. In the inversion point, i.e., for $j_w^v/j_w^a = 1$ the scattering of experimental and theoretical data is about 3%. It's confirmation that the calculation method is suitable for engineering calculations. For the turbulent flow the data coincide with the accuracy of ~10%, and this is also satisfactory result.

Our analytical calculations by formulas (7)–(9) demonstrated that the inversion temperature may be different for different comparison conditions of evaporation intensity into air–steam mix and dry air: this can be the condition of constant mass flow rate $\rho_0 u_0 = \text{const}$ and constant flow velocity $u_0 = \text{const}$ at the same Reynolds number or at the same length. These temperatures for listed conditions of water evaporation into steam/dry air are written down in Table 1.

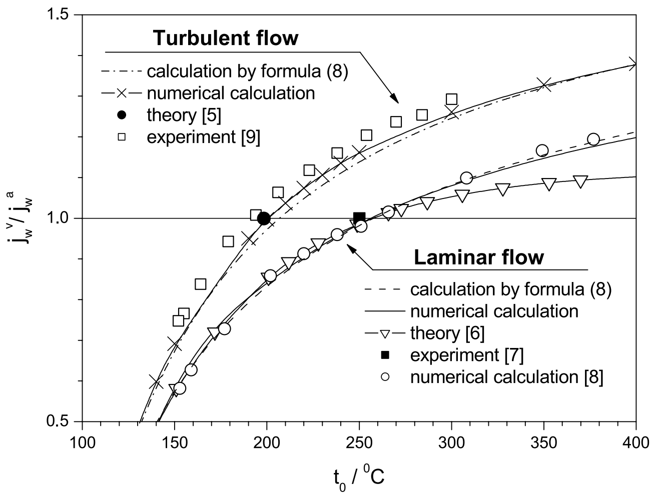

Fig. 4. Inversion temperature at a constant mass flow rate.

Table 1
Inversion temperature for different flow regimes

Flow regime	$Re_x = \text{const}$ $\rho_0 u_0 = \text{const}$	$x = \text{const}$ $\rho_0 u_0 = \text{const}$	$x = \text{const}$ $u_0 = \text{const}$
Laminar	190 °C	257 °C	432 °C
Turbulent	187 °C	207 °C	445 °C

Obviously, at a constant mass flow rate the inversion temperature is smaller for turbulent flow than for laminar; at a constant flow velocity, vise versa, the turbulent flow gives a higher level of inversion temperature. At the equal Reynolds numbers, the temperature is almost independent on the flow regime.

Fig. 5a and b plot the analytical calculations for inversion temperature by formulas 8 and 9 as a function of vapor concentration for drying flow (laminar and turbulent regimes); numerical results are also plotted there. These graphs evidence that at a lower vapor quantity we obtain a lower inversion temperature (for both a constant mass flow rate and a constant volumetric flow rate).

Analysis of formulas (7)–(9) demonstrated that the inversion temperature is not influenced by value of main flow rate, and this is close to numerical experiments at different levels of flow rate ranging from 10 to 50 kg/(m^{2}s), and close to results of [3,5,6].

The authors of papers [4,8,10] assume that the phenomenon of inversion temperature is created a difference between the superheated steam and dry air heat capacities. But calculations on water evaporation into a laminar boundary layer with a different vapor quantity (see Fig. 6) demonstrated that below the inversion point we have a smaller evaporation rate into a flow with a higher vapor concentration (although the heat capacity of an air–vapor mix is higher than for dry air). This means that the difference between the heat capacities of an air–steam and dry air is not the exclusive explanation of this phenomenon.

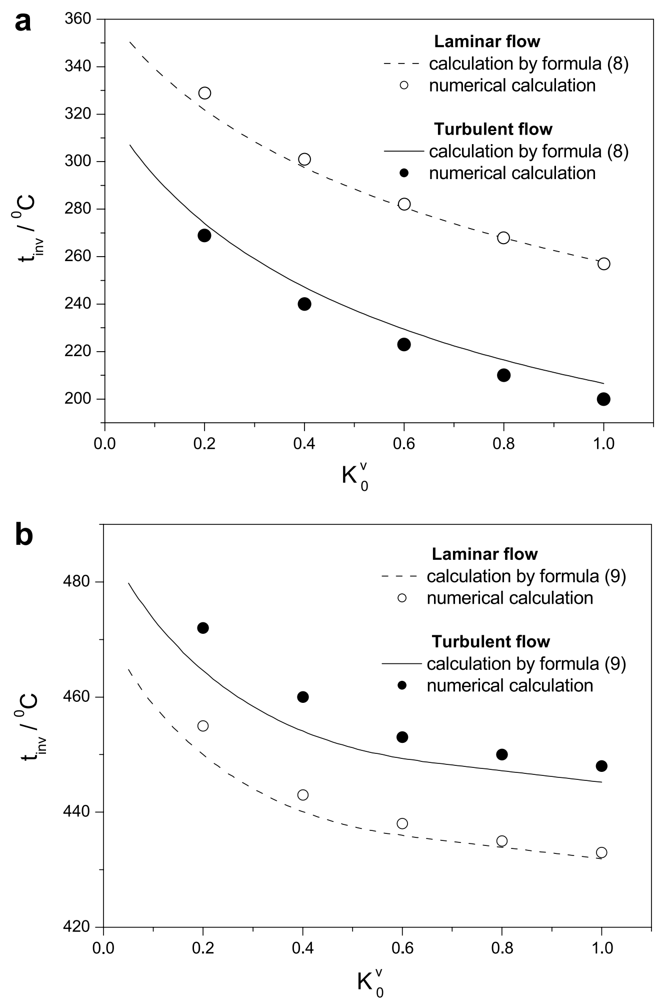

Fig. 5. Inversion temperature vs. vapor concentration in the main flow: (a) constant mass flow rate; (b) constant volumetric flow rate.

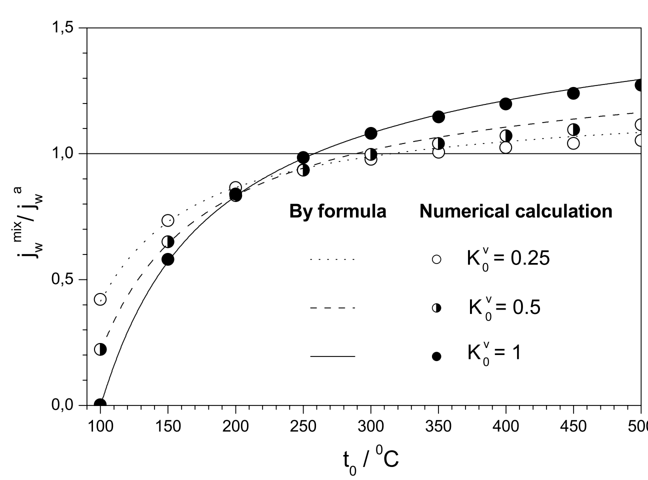

Fig. 6. Ratio for evaporation mass rate of water into an air–steam mixture and dry air.

5. Conclusion

This study has developed the method for calculation of inversion temperature when the water evaporates into an air–steam mix with a constant mass or volume flow rate. The solution is written in a relative form, and this helps to analyze the influence of the different factors (vapor quantity, gas flow rate, flow regime) on the inversion temperature.

If the gas flows with a constant mass flow rate the inversion temperature is twice lower than for the case of constant volumetric flow rate; this is valid both for laminar and turbulent flow regime.

It is interesting that the transition from laminar to turbulent regime leads to increase the inversion temperature at a constant mass flow rate. Opposite at a constant volumetric flow rate the inversion temperature decreases.

Analysis of results on inversion temperature for liquid evaporation into a air–steam mixture with different vapor quantity brings a conclusion that besides all other factors the lower concentration of water vapor in the main stream always increases this kind of temperature.

The approach developed for calculation of inversion temperature can be easily extended to the case of another geometry of evaporation surface and other liquids.

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